Math 407, Midterm 1

10 am on Wed. Mar. 3 until 10 am on Thu. Mar. 4

You may use a calculator (without internet connection or other communication capabilities). You are allowed to look at the textbook, or at the lecture notes from class. However no other sources (for example the internet) are allowed. TO GET FULL CREDIT, WORK MUST BE SHOWN. Upload your solutions to blackboard as a pdf file.

1. Let Y_1 and Y_2 be indicators of independent events with probabilities $1/3$ and $1/4$ respectively.

a) Write down the joint distribution table of $Y_1 + Y_2$ and $Y_1 - Y_2$.

b) Calculate $E(Y_1 - Y_2)$.

c) Calculate the standard deviation of $Y_1 - Y_2$.

2. Toss a fair coin 60 times.

a) Using normal approximation with the continuity correction, estimate the chance that the number of heads is between 29 and 32 inclusive.

b) Repeat a), using normal approximation without the continuity correction.

3. Let π be a random permutation on n symbols. For $1 \leq i \leq n$, say that i is a fixed point of π if $\pi(i) = i$. Let X be the number of fixed points of π.

a) Compute $E(X)$.

b) For large n, approximate the probability that $X = 3$.

4. A box has 498 red and 2 green balls. Let X be the number of greens in 250 random draws with replacement from this box. Calculate

a) $P(X = 1)/P(X = 2)$.

b) $P(X = 1$ given X $= 1$ or $2)$

c) Repeat b) assuming draws without replacement.

Express your answers as a single number, not a complicated expression.