Math 407, Final Exam (Section 39628)

11 am on Fri. May. 7 until 11 am on Sat. May. 8

You may use a calculator (without internet connection or other communication capabilities). You are allowed to look at the textbook, or at the lecture notes from class. However no other sources (for example the internet) are allowed. TO GET FULL CREDIT, WORK MUST BE SHOWN. Upload your solutions to blackboard as a pdf file.

1. Consider the average

$$A_n = (X_1 + X_2 + \cdots + X_n)/n$$

of n independent random variables, each uniformly distributed on $[0, 1]$. Find n so that $P(A_n < 0.51)$ is approximately .80.

2. A random variable has probability density function $f_X(x)$ which is equal to cx^2 if $0 \leq x \leq 2$ and 0 otherwise.

a) Find the constant c

b) Calculate $P(X \leq b)$ for $0 \leq b \leq 2$

c) Calculate $E(X)$

d) Calculate $SD(X)$

3. Let (Y, Z) be a random point uniformly distributed on the triangle with vertices $(0,0), (0,1)$ and $(1,0)$. Calculate the covariance of Y and Z.

4. Suppose we have a medical test which detects a certain disease 97 percent of the time, but gives false positives 2 percent of the time. Assume that .5 percent of the population has the disease. If a random person tests positive for the disease, what is the probability that they actually have the disease ?

5. The joint density of X and Y is given by

$$f(x,y) = \frac{1}{2} y e^{-xy}$$

if $0 < x < \infty, 0 < y < 2$ and $f(x,y) = 0$ otherwise. Calculate $E[e^{X/3}|Y = 1]$.