Midterm

March 19, 2021

1. Let $a, b, c \in \mathbb{N}$. Prove the following using contraposition: if $a^2 + b^2 + c^2 \equiv 3 \mod 4$ then $a + b + c \not\equiv 0 \mod 4$.

2. Show that for any prime number $p > 2$ there is some prime number q strictly greater than p and smaller than $p!$. Prove this claim directly.

3. Use proof by contradiction to show that $n^2 \notin \Omega(n!)$. **You may not use the limit rule. Please use the formal definition of big Omega.**

4. Use proof by induction over $|B|$ to show the following claim: For any sets A and B , $|A \times B| = |A| \times |B|$.

5. Show that $4^{2n-1} + 3^{n+1}$ is divisible by 13 for all $n \geq 1$ by induction.

6. Prove the following claim by contradiction: If p,q and $\sqrt{2}p + \sqrt[3]{3}q$ are all rational number, show that $p = q = 0$. You can assume $\sqrt{2}$ and $\sqrt[3]{3}$ are both irrational.

7. A restaurant has a four course meal and for each course customers have a choice of three dishes per course and must choose exactly one dish. What is the fewest number of customers that the restaurant must have during a single evening dinner service to ensure that 4 customers order the exact same meal i.e. they ordered the same dish for each course for all four courses? Prove your answer.

8. Let $f : X \to Y$ be a function and $f^{-1} : Y \to X$ be its inverse relation. f^{-1} is a bijective function. Show that f is a bijection.

   (a) Use proof by contradiction to show that f is injective.

   (b) Use proof by contradiction to show tht f is surjective.

   (c) You cannot simply cite any results shown in the textbook or class.

   (d) You must write your answer in as much quantificational logic as you can.You will not receive much credit for answers not written in quantificational logic.