1. Let $X_1, \ldots, X_n$ be i.i.d. $N(0, \sigma^2)$ and $Y_1, \ldots, Y_m$ be i.i.d. $N(0, \tau^2)$ and independent of the X_i. For each of the following give the distribution (including the name and values of any parameters) as well as justification.

   (a)
$$\sum_{i=1}^{n} X_i$$

   (b)
$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

   (c)
$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2$$

   (d)
$$\frac{\overline{X}}{S/\sqrt{n}}$$

   (e)
$$\sum_{i=1}^{m} Y_i^2$$

   (f)
$$\frac{\sum_{i=1}^{n} X_i^2}{\sum_{i=1}^{m} Y_i^2}$$

2. Let $X_1, \ldots, X_n$ be i.i.d. with the exponential distribution whose density function is

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x \geq 0, \\ 0, & x < 0, \end{cases} \tag{1}$$

   where $\theta > 0$ is unknown.

   (a) Show that the moment generating function (MGF) of the distribution (1) is

$$M(t) = \frac{\theta}{\theta - t}. \tag{2}$$

   (b) Name an open interval I containing 0 for which (2) is valid for all $t \in I$.

   (c) Use the MGF to compute the first 2 moments of the exponential distribution (1).

(d) Find the Method of Moments estimator $\widehat{\theta}$ of θ.

(e) Use the approximation

$$\frac{1}{\overline{X}} \approx \theta - \theta^2(\overline{X} - 1/\theta) \tag{3}$$

to approximate the variance of $\widehat{\theta}$. Explain how you could use this to approximate the estimated standard error of $\widehat{\theta}$. You can use (3) without justification and assume it is accurate enough for these approximations.