Practice Midterm 2 for MATH 226, section 39559

You have 50 minutes.

Name: **Date:**

Problem	Score
#1	/10
#2	/10
#3	/10
#4	/10
#5	/10
Total	/50

Problem 1 (10 points): Define

$$f(x,y,z) = \frac{1}{x^2} + \frac{4}{y^2} + \frac{9}{z^2}, \tag{1}$$

where $x > 0$, $y > 0$, and $z > 0$. Use Lagrange multipliers to find the minimum value of f on the portion of the surface $x^2 + y^2 + z^2 = 36$ with $x > 0, y > 0, z > 0$. (You may assume that this absolute minimum exists, and that the solution to the system of equations coming from Lagrange multipliers is this absolute minimum.)

Problem 2 (10 points): Consider the function

$$f(x,y) = \frac{1}{5}xy^2 - x \tag{2}$$

on the domain $D = \{(x,y) \mid -2 \le x \le 2,\ x^2 \le y \le 4\}$.

(a; 3 points) Find the critical points of $f(x,y)$ in the interior of D.

(b; 3 points) Classify the critical points of $f(x,y)$ in the interior of D as local minima, local maxima, or saddles.

Problem 2 (continued).

(c; 4 points) Find the absolute maximum and minimum values of $f(x, y)$ on D.

Problem 3 (10 points): Evaluate the following integrals.

(a; 5 points) $I = \displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{y^3+1}\,dy\,dx$ (switch the order!).

(b; 5 points) $I = \displaystyle\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \sqrt{1+x^2+y^2}\,dy\,dx.$

Problem 4 (10 points): Consider the region E that is under the plane $z + 3y = 16$, inside the cylinder $x^2 + y^2 = 4$, and in the first octant.

(a; 3 points) Rewrite the integral $\iiint_E x\, dV$ as an iterated integral in Cartesian coordinates (i.e. x, y, z). (Do not compute this integral.)

(b; 3 points) Rewrite this integral as an iterated integral in cylindrical coordinates.

(c; 4 points) Evaluate this integral.

Problem 5 (10 points):

(a; 5 points) Rewrite the integral $I = \int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x, y, z)\, dz\, dy\, dx$ as an iterated integral in the order $dx\, dy\, dz$. (You may want to draw a picture of the domain of integration, to help you figure out how to switch the order.)

(b; 5 points) Compute the integral $I = \int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^{1} z\, dz\, dx\, dy.$