## Practice Midterm 2 for MATH 226, section 39559

You have 50 minutes.

Name: Date:

| Problem | Score |
|---------|-------|
| #1      | /10   |
| #2      | /10   |
| #3      | /10   |
| #4      | /10   |
| #5      | /10   |
| Total   | /50   |

Problem 1 (10 points): Define

(1) 
$$f(x,y,z) = \frac{1}{x^2} + \frac{4}{y^2} + \frac{9}{z^2},$$

where x > 0, y > 0, and z > 0. Use Lagrange multipliers to find the minimum value of f on the portion of the surface  $x^2 + y^2 + z^2 = 36$  with x > 0, y > 0, z > 0. (You may assume that this absolute minimum exists, and that the solution to the system of equations coming from Lagrange multipliers is this absolute minimum.)

Problem 2 (10 points): Consider the function

(2) 
$$f(x,y) = \frac{1}{5}xy^2 - x$$

on the domain  $D = \{(x, y) \mid -2 \le x \le 2, x^2 \le y \le 4\}.$ 

(a; 3 points) Find the critical points of f(x,y) in the interior of D.

(b; 3 points) Classify the critical points of f(x,y) in the interior of D as local minima, local maxima, or saddles.

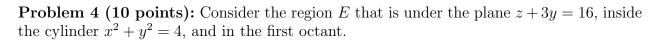
 $\ \, \textbf{Problem 2} \ (\text{continued}).$ 

(c; 4 points) Find the absolute maximum and minimum values of f(x, y) on D.

Problem 3 (10 points): Evaluate the following integrals.

(a; 5 points) 
$$I = \int_0^4 \int_{\sqrt{x}}^2 \frac{3}{y^3 + 1} \, dy \, dx$$
 (switch the order!).

(b; 5 points) 
$$I = \int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \sqrt{1+x^2+y^2} \, dy \, dx$$
.



(a; 3 points) Rewrite the integral  $\iiint_E x \, dV$  as an iterated integral in Cartesian coordinates (i.e. x, y, z). (Do not compute this integral.)

(b; 3 points) Rewrite this integral as an iterated integral in cylindrical coordinates.

(c; 4 points) Evaluate this integral.

## Problem 5 (10 points):

(a; 5 points) Rewrite the integral  $I = \int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x,y,z) \, dz \, dy \, dx$  as an iterated integral in the order  $dx \, dy \, dz$ . (You may want to draw a picture of the domain of integration, to help you figure out how to switch the order.)

**(b; 5 points)** Compute the integral 
$$I = \int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^{1} z \, dz \, dx \, dy$$
.