Practice Midterm 1 for MATH 226, section 39559

You have 50 minutes. You may use one, one-sided sheet of notes. You may not use any calculator, cell phone, or similar device.

Name: **Date:**

Problem	Score
#1	/10
#2	/10
#3	/10
#4	/10
#5	/10
Total	/50

Problem 1: (a) Find an equation for the plane passing through the points $P = (3, 2, 2)$, $Q = (5, -1, 1)$, and $R = (-1, 0, -4)$.

(b) Define C to be the curve C parametrized by $\mathbf{r}(t) = \langle 1 - t^2, t + 1, 2t^2 + t + 2 \rangle$. Find an equation for the line which is tangent to C at $(0, 2, 5)$.

(c) Find the point where the line in (b) intersects the plane in (a).

Problem 2: Consider the curve

$$\mathbf{r}(t) = \left\langle e^t, \frac{\sqrt{2}}{2}e^{2t}, \frac{1}{3}e^{3t} \right\rangle.$$

(a) Compute the length of $\mathbf{r}(t)$, for $0 \leq t \leq 3$.

(b) Suppose that $\rho(s)$ is the reparametrization by arclength of the curve $\mathbf{r}(t)$. Find the length of $\rho(s)$ for $0 \leq s \leq 7$. (Hint: you should not need to do any complicated calculations.)

Problem 3: (a) Define f by

$$f(x,y) = \arctan\left(\log\left(\sqrt{x} + \frac{\cos x}{x^x}\right) - \pi^{1/x}\right) - x^2 y.$$

Compute the partial derivative f_{xxy}. *(Hint: There is a reason that you are only given 1.5".)*

(b) Suppose $u = x^2y^3 + z^4$, where $x = p + 3p^2$, $y = pe^p$, and $z = p\sin p$. Use the chain rule to find u_p.

Problem 4: Let S be the surface in $\mathbb{R}^3$ defined by the equation

$$xz^2 - \arctan(yz) = -\frac{\pi}{4}.$$

(a) Find expressions for $\partial z/\partial x$ and $\partial z/\partial y$. (Recall that $(\arctan u)' = 1/(1+u^2)$.)

(b) Determine whether $(0, 1, 1)$ lies on S. Using linear approximation, find an approximation of the z-coordinate of the point on S that has $x = -0.1$ and $y = 1.1$.

(c) Consider a path $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ lying on S that has $\mathbf{r}(0) = \langle 0, 1, 1\rangle$. Assume that $\frac{\mathrm{d}x}{\mathrm{d}t}(0) = -2$ and $\frac{\mathrm{d}y}{\mathrm{d}t}(0) = 1$. Find the value of $\frac{\mathrm{d}z}{\mathrm{d}t}(0)$.

Problem 5: Let S be the hyperboloid defined by $x^2 + y^2 - z^2 = 1$.

(a) Find an equation of the tangent plane P to S at $(1, 1, 1)$.

(b) Find all points p on S such that the tangent plane to S at p is parallel to P.