Midterm 2 for MATH 226, section 39559

You have 50 minutes.

You may use any resources (textbook, internet, notes, etc.) except that you may not consult any other human.

Show your work! Correct answers with no work may not get any credit.

You may use a calculator to aid your computation, but your final answer should be exact, e.g., π as opposed to 3.14.

Name: **Date:**

Problem	Score
#1	/10
#2	/10
#3	/10
#4	/10
#5	/10
Total	/50

(There is an opportunity for up to 2 points of extra credit on problem 5!)

Problem 1: Consider the function $f(x,y) \coloneqq x^4 + xy + y^4$, with domain the *open* region $D \coloneqq \left\{(x,y)\colon y < x + \frac{1}{2}\right\}$.

(a; 5 points) Find the critical points of f on D.

(b; 5 points) Classify the critical points you found in (a) as local maxima, local minima, or saddles.

Problem 2: Consider the tasks of finding the points on the sphere $x^2 + y^2 + z^2 = 1$ that are at the greatest distance (respectively the least distance) from the point $(0, 1, 2)$.

(a; 5 points) Write down a system of equations, whose solutions are the candidates for the points on this sphere that are the farthest from (respectively, closest to) $(0, 1, 2)$. *(Hint: use Lagrange multipliers, applied to the squared distance function.)*

(b; 5 points) Find these points, and identify which is the farthest from $(0, 1, 2)$ and which is the closest.

Problem 3: (a; 5 points) Change the order of integration and evaluate the following integral:

$$I = \int_0^9 \int_{\sqrt{x}}^3 \cos\big(y^3\big)\, dy\, dx.$$

(b; 5 points) Consider the helix C parametrized by $\mathbf{r}(t) := \big\langle \cos t, \sin t, t \big\rangle$, $0 \le t \le 2\pi$. Compute the following integral:

$$\int_C z\, ds.$$

Problem 4: Consider the region E *below* the cone $z = \sqrt{x^2 + y^2}$, within the cylinder $x^2 + y^2 = 16$, and above the xy-plane. Consider the following integral, which represents the volume of E:

$$I := \iiint_E 1\, dV.$$

(a; 5 points) Rewrite I as an iterated integral in Cartesian coordinates. (You do not have to evaluate this iterated integral.)

(b; 5 points) Rewrite I as an iterated integral in cylindrical coordinates. (You do not have to evaluate this iterated integral.)

Problem 5 (10 points): Consider the following vector field:

$$\mathbf{F}(x,y) := \left\langle y + 2xe^{x^2+y^2}, x + 2ye^{x^2+y^2} \right\rangle.$$

(a; 2 points of extra credit) Without computing a potential function, argue carefully for why $\mathbf{F}$ is conservative.

(b; 5 points) Find a potential function for $\mathbf{F}$, i.e. a function f satisfying $\mathbf{F} = \nabla f$.

(c; 5 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve parametrized by $\mathbf{r}(t) := \left\langle \sqrt{\log t} \cos t, \sqrt{\log t} \sin t \right\rangle$, $\pi \leq t \leq \frac{3\pi}{2}$.