Midterm 1 for MATH 226, section 39559

You have 50 minutes.

You may use any resources (textbook, internet, notes, etc.) except that you may not consult any other human.

Show your work! Correct answers with no work may not get any credit.

Name: **Date:**

Problem	Score
#1	/10
#2	/10
#3	/10
#4	/10
Total	/40

(Note that you can earn up to two points of extra credit in Problem 3, part c!)

Problem 1: Consider

$$f(x, y) = \cos(\pi x^3 + \pi y^2) + \sin(\pi x + \pi y^4),$$

where

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta,$$

with $r \geq 0$ and $\theta \in [0, 2\pi)$.

(a; 5 points) Compute $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ when $r = 2$ and $\theta = \frac{\pi}{3}$. (Recall $\cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.)

(b; 5 points) Approximate, using calculus, the value of f at $r = \frac{2001}{1000}$ and $\theta = \frac{1001\pi}{3000}$. You do not have to write your answer as a single fraction.

Problem 2: Consider the two planes

$$P_1 : x + y - z = 2, \qquad P_2 : 3x - 4y + 5z = 6.$$

(a; 2 points) Find normal vectors $\mathbf{n}_1, \mathbf{n}_2$ to these planes.

(b; 2 points) Let L be the line of intersection of these planes. Find a point on L. (Hint: set one of the coordinates equal to 0; this is the intersection of L with a coordinate plane.)

(c; 4 points) Find a vector equation for L. (Hint: to find a point on both planes, try setting one coordinate to zero, then solving the equations of the planes for x and y.)

(d; 2 points) Find the angle between these planes.

Problem 3: Consider the vector function $\mathbf{r}(t) := \left\langle \frac{2}{3}t^{3/2}, \cos t, \sin t \right\rangle$, $t \geq 0$.

(a; 5 points) Compute the arclength from $t = 0$ to $t = 1$.

(b; 5 points) Compute the reparametrization of $\mathbf{r}(t)$ by arclength, starting from $t = 0$ and in the direction of increasing t.

(c; 2 points of extra credit) Draw a picture of the curve traced out by $\mathbf{r}$, with justification.

Problem 4: A series of small questions. Make sure to justify, in a sentence or two.

(a; 2 points) True or false: the cross product of two 3-dimensional vectors is a scalar.

(b; 2 points) True or false: If $\mathbf{v}, \mathbf{w}$ are any vectors of the same dimension, then $\mathbf{v} \cdot \mathbf{w}$ makes sense and is zero exactly when $\mathbf{v}$ and $\mathbf{w}$ are orthogonal (i.e., perpendicular).

(c; 2 points) True or false: the z-slices of the quadric surface $x^2 + y - 2z^2 = 1$ are ellipses.

(d; 2 points) Suppose that we are standing at the base of a mountain. We are standing at the origin $(0,0)$, and the mountain is located at $(0,1)$ (due north). Let $E(x,y)$ denote the elevation at the point (x,y). What can you say about the sign of $\frac{\partial E}{\partial y}(0,0)$?

(e; 2 points) True or false: if $f(x,y) = x^2$, then $\nabla f(x,y) = 2x$.