Some Things to Keep in Mind

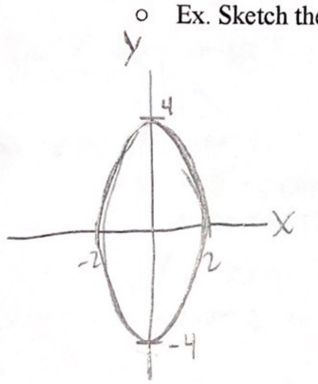
- Two vectors are orthogonal iff $\mathbf{v} \cdot \mathbf{w} = 0$
- Two vectors are parallel if $\mathbf{v}$ is a scalar multiple of $\mathbf{w}$ (or $\mathbf{v} \times \mathbf{w} = 0$)
- $\mathbf{v} \times \mathbf{w}$ is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$
- Three vectors are coplanar iff their triple product is 0
- To write the equation for a line, you need a **point** and a **direction** vector
- To write the equation for a plane, you need a **point** and a **normal** vector

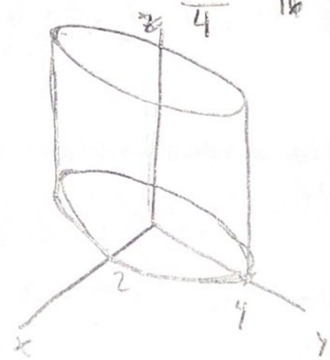
Tips for Parameterizing Curves

- Rewrite equations isolating one variable in terms of another, then substitute for t
- Polar coordinates are common for shapes like ellipses and circles ($x = a\,cos(t)$, $y = b\,sin(t)$)
- Curves can be parametrized in an infinite number of ways, but some will make your calculations easier than others

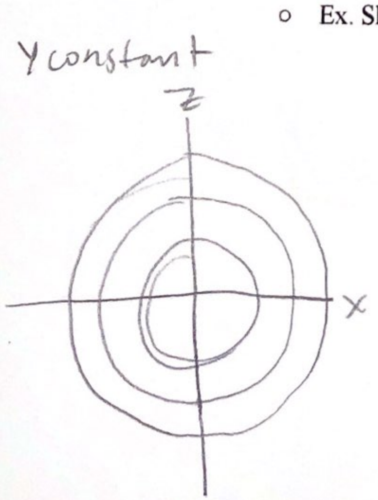
Sketching Quadric Surfaces

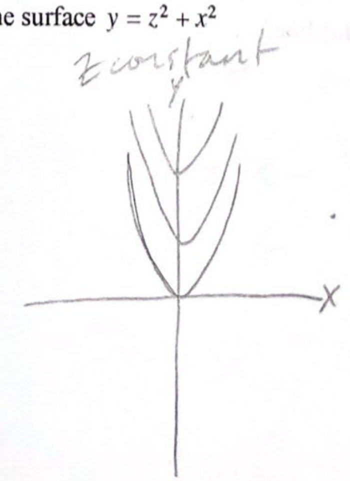
- If the equation only has two variables, draw the curve on those two axes, and then stretch the image along the remaining axis to obtain the 3D shape.
  - Ex. Sketch the surface $4x^2 + y^2 = 16$ $\rightarrow$ $\frac{x^2}{4} + \frac{y^2}{16} = 1$

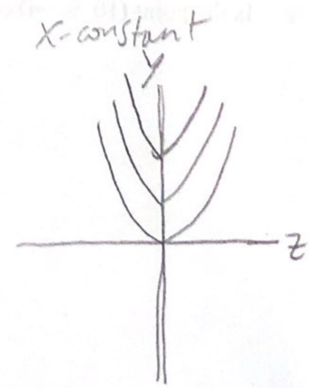


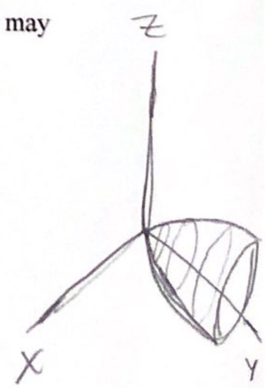


- Draw traces of the graph by holding one variable constant and graphing the remaining two variables in two dimensions. Repeat until you can determine the shape of the graph. You may need to change which variable is held constant to get a clear picture of the shape.
  - Ex. Sketch the surface $y = z^2 + x^2$









Practice Problems

1. UConn Fall 2008 Midterm 1 Q1

   a. Give a vector equation of the line containing the points (1, −3, 2) and (4, 1, 0).

$\vec{r}_0 = \langle 1,3,2\rangle \quad \vec{v} = \langle 4,1,0\rangle - \langle 1,-3,2\rangle = \langle 3,4,-2\rangle$

$\vec{r}(t) = \vec{r}_0 + t\vec{v}$

$\boxed{\vec{r}(t) = \langle 1,-3,2\rangle + t\langle 3,4,-2\rangle}$

   b. Find the intersection of the line with the plane $x + y + z = 10$.

$\vec{r}(t) = \langle 1,-3,2\rangle + t\langle 3,4,-2\rangle \Rightarrow \langle 3t+1, 4t-3, -2t+2\rangle$

plug into x, y, z

$(3t+1)+(4t-3)+(-2t+2)=10$

$5t = 10$

$t = 2$

$\vec{r}(2) = \langle 3(2)+1, 4(2)-3, -2(2)+2\rangle = \langle 7,5,-2\rangle \rightarrow \boxed{(7,5,-2)}$

   c. Is the point (10, 9, −4) on this line?

$3t+1=10 \rightarrow t=3$

$4t-3=9 \rightarrow t=3$

$-2t+2=-4 \rightarrow t=3$

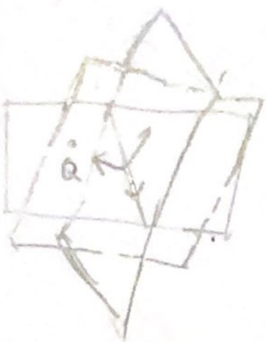
$\boxed{\text{yes}}$

2. USC Fall 2016 Final Q1

Consider two planes, P_1: $x + y = 27$ and P_2: $2x + z = 10$, and a point $Q = (3, 4, 1)$.

a. Write an equation of the plane that passes through the point Q and is perpendicular to the planes P_1 and P_2.

need normal vector of plane.
cross normal vectors of P_1 and P_2



$\vec{n}_1 = \langle 1, 1, 0 \rangle$ $\quad \vec{n}_1 \times \vec{n}_2 = \langle 1, -1, -2 \rangle$

$\vec{n}_2 = \langle 2, 0, 1 \rangle$

$$\boxed{(x-3)-(y-4)-2(z-1)=0} \leftarrow P_3$$

b. Write a parametric equation of the line that passes through the point Q and is parallel to the planes P_1 and P_2.

direction parallel to P_1 and P_2 is the same as the normal vector of P_3

$\vec{v} = \langle 1, -1, -2 \rangle$

$$\boxed{x = 3+t \quad y = 4-t \quad z = 1-2t}$$

3. UConn Fall 2008 Midterm 1 Q2

For the curve $\mathbf{r}(t) = \langle 2t^3,\ 1 - t^3,\ -2t^3 \rangle$, $t \geq 0$:

a. Reparametrize the curve with respect to arc-length, starting at $t = 0$ and moving in direction of increasing t.

$$\vec{r}\,'(t) = \langle 6t^2, -3t^2, -6t^2 \rangle = 3t^2\langle 2,-1,2 \rangle$$

$$|\vec{r}\,'(t)| = 3t^2\sqrt{4+1+4} = 9t^2$$

$$s(t) = \int_0^t 9u^2\,du$$

$$s = 3u^3\Big|_0^t = 3t^3 \Rightarrow t^3 = \frac{s}{3}$$

$$\boxed{\vec{r}(s) = \left\langle \frac{2s}{3},\ 1 - \frac{s}{3},\ \frac{-2s}{3} \right\rangle}$$

b. Find the distance along the curve from $(2, 0, -2)$ to $(16, -7, \overset{-16}{\cancel{16}})$.

$$\left.\begin{aligned} 2t^3 &= 2 \\ 1 - t^3 &= 0 \\ -2t^3 &= -2 \end{aligned}\right\} t = 1 \qquad \left.\begin{aligned} 2t^3 &= 16 \\ 1 - t^3 &= -7 \\ -2t^3 &= -16 \end{aligned}\right\} t = 2$$

$$\int_1^2 |\vec{r}\,'(t)|\,dt = \int_1^2 9t^2\,dt = 3t^3\Big|_1^2 = 3(8-1) = 21$$

4. Colgate Spring 2003 Practice Exam 1 Q7

Let $\mathbf{v} = 2\,\mathbf{i} + a\,\mathbf{j} + a^2\,\mathbf{k}$ and $\mathbf{w} = (2a-3)\,\mathbf{i} + \mathbf{j} + \mathbf{k}$:

a. For which values of a are the vectors perpendicular?

2 vectors are perpendicular if $\vec{v} \cdot \vec{w} = 0$

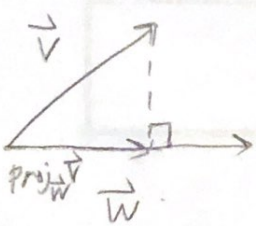
$$\langle 2, a, a^2 \rangle \cdot \langle 2a-3, 1, 1 \rangle = 4a - 6 + a + a^2 = 0$$

$$a^2 + 5a - 6 = 0$$

$$(a+6)(a-1) = 0$$

$$\boxed{\begin{array}{l} a = -6 \\ a = 1 \end{array}}$$

b. Write $\mathbf{v}$ as the sum of a vector parallel to $\mathbf{w}$ and a vector perpendicular to $\mathbf{w}$ when $a = 0$ (Hint: use vector projection!).



$$\text{proj}_{\vec{w}} \vec{v} = \frac{\vec{w} \cdot \vec{v}}{|\vec{w}|^2} \vec{w}$$

$$\vec{v} = \langle 2, 0, 0 \rangle \qquad \vec{w} = \langle -3, 1, 1 \rangle \qquad |\vec{w}| = \sqrt{9+1+1} = \sqrt{11}$$

$$\text{proj}_{\vec{w}} \vec{v} = \frac{-6}{(\sqrt{11})^2} \langle -3, 1, 1 \rangle$$

$$= \frac{-6}{11} \langle -3, 1, 1 \rangle$$

$$\langle 2, 0, 0 \rangle = \frac{-6}{11} \langle -3, 1, 1 \rangle + \vec{u}$$

$$\vec{u} = \langle 2, 0, 0 \rangle - \langle \tfrac{18}{11}, -\tfrac{6}{11}, \tfrac{-6}{11} \rangle$$

$$\vec{u} = \langle \tfrac{4}{11}, \tfrac{6}{11}, \tfrac{6}{11} \rangle$$

$$\boxed{\vec{v} = \langle \tfrac{18}{11}, \tfrac{-6}{11}, \tfrac{-6}{11} \rangle + \langle \tfrac{4}{11}, \tfrac{6}{11}, \tfrac{6}{11} \rangle}$$

Practice Problems

5. Colgate Spring 2003 Practice Exam 2 Q10
   Suppose f is a differentiable function such that

$$f(1,3) = 1, \quad f_x(1,3) = 2, \text{ and } f_y(1,3) = 4.$$

   a. Find a vector in the plane that is perpendicular to the contour line $f(x, y) = 1$ at the point (1, 3).

gradient is perpendicular to contour line

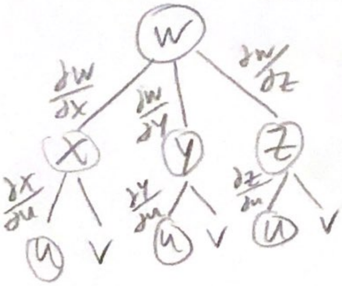
$$\nabla f = \langle f_x, f_y \rangle = \boxed{\langle 2, 4 \rangle}$$

   b. At the point (1, 3), what is the rate of change of f in the direction $\mathbf{i} + \mathbf{j}$?

$$\vec{u} = \frac{\langle 1, 1 \rangle}{\sqrt{2}} \qquad D_{\vec{u}} f = \nabla f \cdot \vec{u}$$

$$\langle 2, 4 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \frac{2}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \boxed{\frac{6}{\sqrt{2}}}$$

6. MIT OpenCourseWare
   Let $w = xyz$, $x = u^2 v$, $y = uv^2$, $z = u^2 + v^2$. Find $\frac{\partial w}{\partial u}$ at the point $(u, v) = (1, 2)$.



$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$x(1,2) = 2 \qquad y(1,2) = 4 \qquad z(1,2) = 5$$

$$\frac{\partial w}{\partial x} = yz = 20 \qquad \frac{\partial w}{\partial y} = xz = 10 \qquad \frac{\partial w}{\partial z} = xy = 8$$

$$\frac{\partial x}{\partial u} = 2uv = 4 \qquad \frac{\partial y}{\partial u} = v^2 = 4 \qquad \frac{\partial z}{\partial u} = 2u = 2$$

$$\frac{\partial w}{\partial u} = 20(4) + 10(4) + 8(2) = 80 + 40 + 16$$

$$= \boxed{136}$$

7. UConn Fall 2008 Midterm 2 Q2
Consider the surface

$$2x^2y^5 + (zx)^2 - x^4yz = 16$$

a. Find the equation of the tangent plane to this surface at the point (2, 0, 2).

$F(x,y,z) = 2x^2y^5 + (zx)^2 - x^4yz$

$\nabla F = \langle 4xy^5 + 2xz^2 - 4x^3yz,\ 10x^2y^4 - x^4z,\ 2x^2z - x^4y \rangle$

$\nabla f(2,0,2) = \langle 0+16+0,\ 0-32,\ 16-0 \rangle = \langle 16,-32,16 \rangle$
$= 16\langle 1,-2,1 \rangle$

$\boxed{(x-2)-2y+(z-2)=0}$

b. Approximate the y value of the point on the surface where $x = 2.03$ and $z = 1.99$.

$(2.03-2)-2y+(1.99-2)=0$

$0.03-2y-0.01=0$

$0.02=2y$

$\boxed{y=0.01}$

8. MIT OpenCourseWare
Find the maximum and minimum values of $f(x, y) = x^2 + x + 2y^2$ on the unit circle.

$g(x,y) = x^2+y^2=1$

$\nabla f = \langle 2x+1, 4y \rangle \quad \nabla g = \langle 2x, 2y \rangle$

$\nabla f = \lambda \nabla g \qquad 4y = 2y\lambda$

$2x+1=2x\lambda \qquad$ if $y \neq 0$: $\lambda = 2$

$2x-2x\lambda=-1 \qquad x=\frac{1}{2} \quad y=\pm\sqrt{1-(\frac{1}{2})^2} = \pm\frac{\sqrt{3}}{2}$

$x(2-2\lambda)=-1 \qquad$ if $y=0$: $x=\pm 1$

$x=\frac{-1}{2-2\lambda}$

points: $(\frac{1}{2},\pm\frac{\sqrt{3}}{2}),\ (1,0),\ (-1,0)$

value: $\frac{9}{4},\ 2,\ 0$

$\boxed{\begin{array}{l} \text{max: } f(\frac{1}{2},\pm\frac{\sqrt{3}}{2}) = \frac{9}{4} \\ \text{min: } f(-1,0)=0 \end{array}}$

9. USC Spring 2015 Final Q3

Consider the function f that is given by

$$f(x, y) = y^3 - 2xy + x^2$$

a. Find all the critical points of f.

$\nabla f = \vec{0}$

$\nabla f = \langle -2y+2x,\ 3y^2 - 2x \rangle = \vec{0}$

$\begin{cases} -2y+2x = 0 \\ 3y^2 - 2x = 0 \end{cases}$

$3y^2 - 2y = 0$

$y(3y-2) = 0$

$y = 0 \rightarrow x = 0$

$y = \frac{2}{3} \rightarrow x = \frac{2}{3}$

$\boxed{(0,0) \quad \left(\frac{2}{3}, \frac{2}{3}\right)}$

b. Classify each critical point you found above (if you can) as a local maximum, local minimum, or saddle point.

$f_{xx} = 2 \quad f_{xy} = -2 \quad f_{yy} = 6y$

$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$

$D = 12y - 4$

$D(0,0) = -4$

$f(0,0)$ is a saddle point

$D\left(\frac{2}{3}, \frac{2}{3}\right) = 8 - 4 = 4$

$f\left(\frac{2}{3}, \frac{2}{3}\right)$ is a local min

c. Does f have a *global* maximum on the plane? If it does, state its value and where it is attained. If not, explain why not.

no, neither critical point is classified as a maximum and function increases infinitely

d. Does f have a *global* minimum on the plane? If it does, state its value and where it is attained. If not, explain why not.

no, function only has a local min, as it decreases infinitely

FinalReview Key

Monday, November 16, 2020 1:04 AM

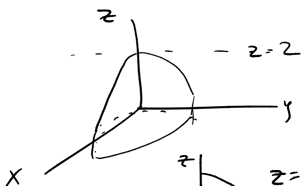
MATH 226/229

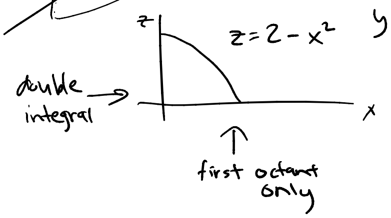
Final Exam Review SI Leaders: Kaylee and Bryson

Let E be the region consisting of those points (x, y, z) in the first octant (where $x \geq 0, y \geq 0, z \geq 0$) and under the paraboloid of equation ~~[illegible]~~. Find the bounds of integration in the following order:

$z = 2 - x^2 - 2y^2$

$$\iiint_E f(x,y,z) = \iiint f(x,y,z)\, \underline{dydxdz}$$





$z \to [0, 2]$

$x \to [\cancel{-\sqrt{2-z}}\ 0, \sqrt{2-z}]$

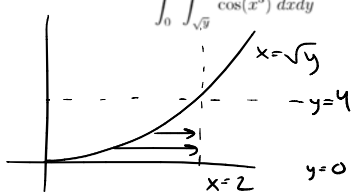
$y \to [\cancel{-\sqrt{1-\frac{1}{2}x^2-\frac{1}{2}z}}\ 0, \sqrt{1-\frac{1}{2}x^2-\frac{1}{2}z}]$

$$\int_0^2 \int_0^{\sqrt{2-z}} \int_0^{\sqrt{1-\frac{1}{2}x^2-\frac{1}{2}z}} f(x,y,z)\, dydxdz$$

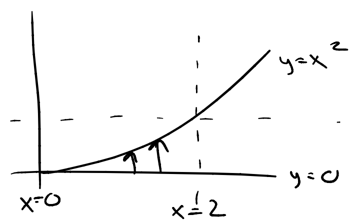
Evaluate the following integral:

$$\int_0^4 \int_{\sqrt{y}}^2 \cos(x^3)\, dxdy$$

Type II



Type I



$$\int_0^2 \int_0^{x^2} \cos(x^3)\, dydx$$

$$= \int_0^2 x^2 \cos(x^3)\, dy\, dx$$

$$= \frac{1}{3}\int_0^8 \cos(u)\, du = \boxed{\frac{1}{3}\sin(8)}$$

MATH 226/229

Final Exam Review SI Leaders: Kaylee and Bryson

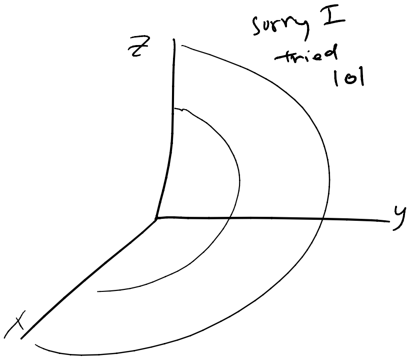
Evaluate the following integral:

$$\iiint_E x^2\, dV$$

where E is the solid bounded by $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$ and the xz plane.

hemispheres, solid bounded on both sides

sorry I tried lol



$x^2 + y^2 + z^2 = 9 \to \rho = 3$

$x^2 + y^2 + z^2 = 16 \to \rho = 4$

$\phi \to [0, \pi] \to$ positive to negative z-axis

$\theta \to [0, \pi] \to$ bounded by xz plane

x^2

$\int^{\pi} \int^{\pi} \int^{4} (\rho^2 \ldots)(\rho^2 \sin\phi)\, d\rho\, d\phi\, d\theta$

$\theta \to [0, \pi] \to$ bounded by xz plane

$$\int_0^{\pi}\int_0^{\pi}\int_3^4 \left(\rho^2 \sin^2\phi \cos^2\theta\right)\left(\rho^2 \sin\phi\right) d\rho\, d\phi\, d\theta$$

↳ Jacobian

$$\frac{4^5-3^5}{5}\int_0^{\pi}\int_0^{\pi} \sin^3\phi \cos^2\theta \, d\phi\, d\theta$$

$\sin^3\phi \to (1-\cos^2\phi)\sin\phi$

$$= \frac{4^5-3^5}{5}\int_0^{\pi} \cos^2\theta\, d\theta \int_0^{\pi} \sin\phi - \sin\phi\cos^2\phi \, d\phi$$

$$= \frac{4^5-3^5}{5}\int_0^{\pi} \frac{1+\cos 2\theta}{2}\, d\theta \left[\int_0^{\pi} \sin\phi\, d\phi - \int_0^{\pi} \sin\phi\cos^2\phi\, d\phi\right.$$

$u = \cos\phi$, $du = -\sin\phi$

$$= \frac{4^5-3^5}{10}\left[\theta + \tfrac{1}{2}\sin 2\theta\right]_0^{\pi}\left(\left[-\cos\phi\right]_0^{\pi} + \left[\tfrac{1}{3}u^3\right]_1^{-1}\right)$$

$$= \frac{4^5-3^5}{10}(\pi)\left(2-\tfrac{1}{3}-\tfrac{1}{3}\right)$$

$$= \boxed{\frac{4^5-3^5}{10}\left(\frac{4}{3}\pi\right)}$$

Suppose a particle of mass M is located at the origin and a particle of mass m is located at the point (x, y, z). The gravitational force between the two particles is given by

$$\mathbf{F} = < \underbrace{\frac{-mMgx}{(x^2+y^2+z^2)^{3/2}}}_{P}, \underbrace{\frac{-mMgy}{(x^2+y^2+z^2)^{3/2}}}_{Q}, \underbrace{\frac{-mMgz}{(x^2+y^2+z^2)^{3/2}}}_{R} >$$

- Show that $\mathbf{F}$ is conservative
- Find the potential function of $\mathbf{F}$
- A second particle moves from (1,2,3) to (2,1,0) along the curve C parameterized by

$$\mathbf{r}(t) = < t + e^t, 1 + \sin(\frac{\pi t}{2}), 3 - 3t >$$

for $0 \le t \le 1$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$

a.) show curl $\vec{F} = 0 \to R_y = Q_z,\ P_z = R_x,\ Q_x = P_y$

$$R_y = \frac{3}{2} mMgz(2y)(x^2+y^2+z^2)^{-5/2}$$
$$Q_z = \frac{3}{2} mMgy(2z)(x^2+y^2+z^2)^{-5/2} \quad ✓$$

$$P_z = \frac{3}{2} mMgx(2z)(x^2+y^2+z^2)^{-5/2}$$
$$R_x = \frac{3}{2} mMgz(2x)(x^2+y^2+z^2)^{-5/2} \quad ✓$$

$$P_y = \frac{3}{2} mMgx(2y)(x^2+y^2+z^2)^{-5/2}$$
$$Q_x = \frac{3}{2} mMgy(2x)(x^2+y^2+z^2)^{-5/2} \quad ✓$$

b.) $f_x = P,\ f_y = Q,\ f_z = R$

$f(x,y,z) = \int P\, dx \to u = x^2+y^2+z^2,\ du = 2x\,dx$

$$= -\frac{1}{2}\int \frac{mMg}{u^{3/2}}\, du = \frac{mMg}{\sqrt{x^2+y^2+z^2}} + g(y,z)$$

$f_y = -\frac{1}{2} mMg(x^2+y^2+z^2)^{-3/2}(2y) + g'(y,z) \overset{?}{=} Q \to g'(y,z) = 0,\ g(y,z) = K$

$$f(x,y,z) = \frac{mMg}{\sqrt{x^2+y^2+z^2}} + h(z)$$

$f_z = -\frac{1}{2} mMg(2z)(x^2+y^2+z^2)^{-3/2} + h'(z) \overset{?}{=} R \to h'(z) = 0,\ h(z) = K$

$$\boxed{f(x,y,z) = \frac{mMg}{\sqrt{x^2+y^2+z^2}} + K}$$

c.) FTC for line integrals

b/c $\vec{F}$ is conservative use potential function f

Evaluate:

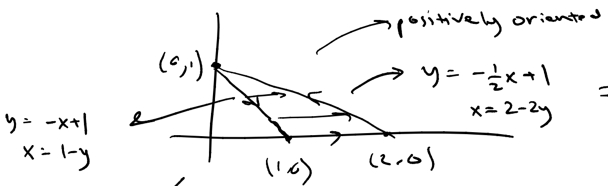
$$\int_C \vec{F}\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$
$$= f(2,1,0) - f(1,2,3)$$
$$= \frac{mMg}{\sqrt{5}} + K - \left(\frac{mMg}{\sqrt{14}} + K\right)$$
$$= \boxed{\frac{mMg}{\sqrt{5}} - \frac{mMg}{\sqrt{14}}}$$

Evaluate

$$\oint_C \overbrace{(x^4)}^{}_{P}dx + \overbrace{(xy)}^{}_{Q}dy$$ closed path C

where C is given by the lines from (1,0) to (2,0), from (2,0) to (0,1), and from (0,1) to (1,0).

↳ Green's Thm



↳ evaluate as Type II ($dx\,dy$),

if you use Type I then you'll have to write ~~two~~ integrals (diff y-bounds at (1,0))

$$\oint_C P\,dx + Q\,dy \overset{\text{Green's}}{=} \iint_D (Q_x - P_y)\,dA$$

$$= \int_0^1 \int_{1-y}^{2-2y} (Q_x - P_y)\,dx\,dy$$

$Q_x = y,\ P_y = 0$

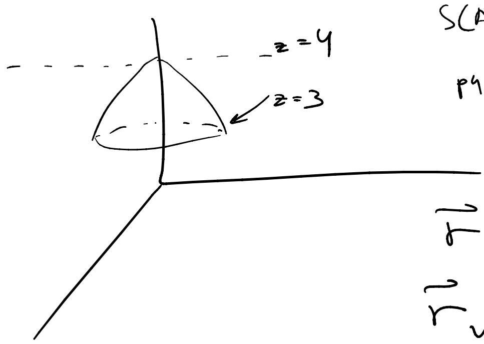
$$\int_0^1 \int_{1-y}^{2-2y} y\,dx\,dy$$

$$\int_0^1 2y - 2y^2 - y + y^2\,dy$$

$$= \int_0^1 y - y^2\,dy$$

$$= \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$$

Consider the surface S with upwards orientation (positive) defined as the open paraboloid $z = 4 - x^2 - y^2$ for $3 \le z \le 4$. Find the surface area of S.



$$S(A) = \iint_D |\vec{r}_u \times \vec{r}_v|\,dA$$

parameterize
u as the radius r
v as the angle θ] polar

$$\vec{r}(u,v) = \langle \overbrace{u\cos v}^{x}, \overbrace{u\sin v}^{y}, \overbrace{4-u^2}^{z} \rangle$$

$$\vec{r}_u = \langle \cos v, \sin v, -2u \rangle$$

$$\vec{r}_v = \langle -u\sin v, u\cos v, 0 \rangle$$

$0 \le v \le 2\pi$

$0 \le u \le 1$ ↳ why 1?

$u=0$ at $z=4$

$u=1$ at $z=3$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos v & \sin v & -2u \\ -u\sin v & u\cos v & 0 \end{vmatrix} = 2u^2\cos v\,\hat{i} + 2u^2\sin v\,\hat{j}$$

$u=1$ at $z=5$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos v & \sin v & -2u \\ -u\sin v & u\cos v & 0 \end{vmatrix} = 2u^2\cos v\,\hat{i} + 2u^2 \sin v\,\hat{j} + u\hat{k}$$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{4u^4\cos^2 v + 4u^4 \sin^2 v + u^2} = u\sqrt{4u^2+1}$$

$$S(A) = \int_0^{2\pi}\int_0^1 u\sqrt{4u^2+1}\, du\, dv$$

let $w = 4u^2+1$, $dw = 8u\,du$

$$= \frac{\pi}{4}\int_1^5 \sqrt{w}\, dw = \frac{\pi}{4}\left(\frac{2}{3}\right)\left[5^{3/2} - 1^{3/2}\right] = \boxed{\frac{(5\sqrt{5}-1)\pi}{6}}$$

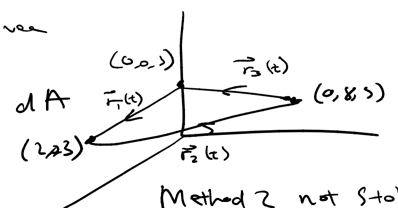
Use any method to find the line integral:

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

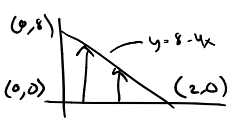
where $\mathbf{F} = y\,\mathbf{i} + x^2\,\mathbf{j} + xz^3\,\mathbf{k}$, and C is the triangle with vertices (0, 0, 3), (2, 0, 3) and (0, 8, 3), oriented by the ordering of the points.

Method 1 Stokes'

$$\oint \vec{F} \cdot d\vec{r} = \iint_D \operatorname{curl} \vec{F} \cdot \underbrace{\vec{N}}_{\text{normal vec}}\, dA$$



$\operatorname{curl}\vec{F} = 2x-1$, $\vec{N} = \langle 0,0,1\rangle$ (flat surface)



$$\int_0^2\int_0^{8-4x} (2x-1)\, dy\, dx$$

$$= \int_0^2 16x - 8 - 8x^2 + 4x\, dx$$

$$= 4\int_0^2 5x - 2 - 2x^2\, dx$$

$$= 4\left[\frac{5}{2}(4) - 2(2) - \frac{2}{3}(8)\right]$$

$$= 4\left[10 - 4 - \frac{16}{3}\right]$$

$$= \boxed{\frac{8}{3}}$$

Method 2 not Stokes' (verify Stokes' by getting same answer)

parameterize all 3 curves using $\vec{r}(t) = (1-t)\langle x_0, y_0, z_0\rangle + t\langle x, y, z\rangle$ for $0 \le t \le 1$

① $\vec{r}_1(t) = (1-t)\langle 0,0,3\rangle + t\langle 2,0,3\rangle = \langle 2t, 0, 3\rangle$

plug in

$\vec{F}_1(t) = \langle 0, 4t^2, 54t\rangle$

② $\vec{r}_2(t) = \langle 2-2t, 8t, 3\rangle$

$\vec{F}_2(t) = \langle 8t, (2-2t)^2, 27(2-2t)\rangle$

③ $\vec{r}_3(t) = \langle 0, 8-8t, 3\rangle$

$\vec{F}_3(t) = \langle 8-8t, 0, 0\rangle$

$$\oint \vec{F}\cdot d\vec{r} = \int_0^1 \vec{F}_1 \cdot \vec{r}_1{}'(t)\, dt + \int_0^1 \vec{F}_2 \cdot \vec{r}_2{}'(t)\, dt + \int_0^1 \vec{F}_3 \cdot \vec{r}_3{}'(t)\, dt$$

$$= \int_0^1 0\, dt + \int_0^1 -16t + 8(2-2t)^2\, dt + \int_0^1 0\, dt$$

$$= \int_0^1 -16t + 32 - 64t + 32t^2\, dt$$

$$= 16\left[\frac{2}{3} - \frac{5}{2} + 2\right]$$

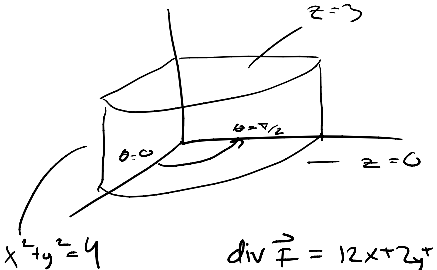
$$= \boxed{\frac{8}{3}}$$

Let E be the three-dimensional region cut from the first octant by the cylinder $x^2+y^2=4$ and the plane $z=3$, and let S be the bounding surface of E. The vector field $\mathbf{F}$ is:

$$\mathbf{F} = \overset{P}{(6x^2+2xy)}\,\mathbf{i} + \overset{Q}{(2y+x^2z)}\,\mathbf{j} + \overset{R}{(4x^2y^3)}\,\mathbf{k}$$

Find the flux of $\mathbf{F}$ out of the surface S:

$$\iint_S \mathbf{F}\cdot d\mathbf{S}$$



div Thm

$$\iint_S \vec{F}\cdot d\vec{S} \overset{div}{=} \iiint_E \operatorname{div}\vec{F}\, dV$$

$$= \int_0^{\pi/2}\int_0^2\int_0^3 \operatorname{div}\vec{F}\, r\, dz\, dr\, d\theta$$

$$\operatorname{div}\vec{F} = 12x+2y+2$$

Jacobian

$$= 2\int_0^{\pi/2}\int_0^2\int_0^3 (6r\cos\theta + r\sin\theta + 1)\,\underline{r}\, dz\, dr\, d\theta$$

$$= 6\int_0^{\pi/2}\int_0^2 6r^2\cos\theta + r^2\sin\theta + r\, dr\, d\theta$$

$$= 6\int_0^{\pi/2} 2(8)\cos\theta + \frac{1}{3}(8)\sin\theta + \frac{1}{2}(4)\, d\theta$$

$$= 24\left[4\sin\theta - \frac{2}{3}\cos\theta + \frac{1}{2}\theta\right]_0^{\pi/2}$$

$$= 24\left(4+\frac{\pi}{4}+\frac{2}{3}\right)$$

$$= \boxed{112+6\pi}$$

Good luck with the final exam, you got this!!

Food place of the week! - Din Tai Fung!