Homework 1 for MATH 226, due Friday, August 28, 2020

(all numbers indicate exercises from the textbook)

§10.1: 6, 8, 10, 12, 14, 28, 32

§10.2: 2, 4, 6, 10, 14, 18, 20, 34

§10.3: 6, 12, 18, 20, 22, 24, 30, 32, 38

optional challenge problem: §10.1.36

Homework 2 for MATH 226, due Friday, September 4, 2020

(all numbers indicate exercises from the textbook)

§10.4: 6, 14, 28, 30, 43, 44

§10.5: 2, 4, 10, 20, 27, 42

§10.6: 8, 10, 12, 14, 32

Homework 3 for MATH 226, due Friday, September 11, 2020

(all numbers indicate exercises from the textbook)

§10.7: 6, 12, 17–22, 28, 30, 36, 46, 54, 60

§10.8: 4, 9, 12

Homework 4 for MATH 226, due Friday, September 18, 2020

(all numbers indicate exercises from the textbook)

§11.1: 18, 20, 22, 24, 26, 30, 41–46

§11.3: 4, 8, 32, 42, 60, 74

§11.4: 4, 20, 28, 34

Homework 5 for MATH 226, due Friday, September 25, 2020

(all numbers indicate exercises from the textbook)

§11.5: 4, 6, 14, 18, 30, 32

§11.6: 4, 6, 8, 16, 22, 34

Homework 6 for MATH 226, due October 9, 2020

(all numbers indicate exercises from the textbook)

§11.7: 6, 26, 36, 42

§11.8: 4, 22, optional: 28

§12.1: 10, 16, 24

Additional problems:

1. Consider the function $f(x, y) = x^4 + y^4 - 4xy$.

(a) Identify and classify all critical points of the function f.

(b) Determine the minimum and maximum values of the function f on the curve $x^4 + y^4 = 32$. *(You can either parametrize this curve or use Lagrange multipliers. Try it both ways, if you're feeling intrepid.)*

(c) Determine the absolute minimum and maximum values of the function f on the region $x^4 + y^4 \leq 32$.

2. Use Lagrange multipliers to find the absolute maximum and minimum values of the function $f(x, y) = x^2 y$ subject to the constraint $3x^2 + 2y^2 = 4$.

3. Recall that the average value of a function $f(x, y)$ on a rectangle R is defined to be the quantity

$$\frac{1}{A(R)} \iint_R f(x, y)\, dx\, dy,$$

where $A(R)$ denotes the area of R. Compute the average value of the function $f(x, y) = y \sin(xy)$ on the rectangle $\{0 \leq x \leq 1, 0 \leq y \leq \pi\}$.

Homework 7 for MATH 226, due October 18, 2020

(all numbers indicate exercises from the textbook; three additional problems will be added by Sunday, October 11)

§12.2: 2, 14, 18, 24, 46

§12.3: 6, 8, 16, 24

§12.5: 4, 14, 26

Additional problems:

1. Change the order of integration in the following integral. You should get a sum of two integrals. You don't need to evaluate these integrals.

$$\int_0^4 \int_x^{2x} e^y \, dy \, dx.$$

(Optional: evaluate these integrals.)

2. Use polar coordinates to compute the volume below the surface

$$z = \frac{3x^2 + 4y^2}{\sqrt{x^2 + y^2}}$$

and above the disk $x^2 + y^2 \leq 1$ in the xy-plane. *Hint: use the half-angle formula to rewrite* $\sin^2 \theta$ *as* $\frac{1-\cos 2\theta}{2}$.

3. Consider the solid E bounded by the tetrahedron with corners at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 0, 2)$, and $(0, 2, 2)$. If $f(x, y, z)$ is any continuous function, write $\iiint_E f(x, y, z)\, dV$ as an iterated integral, with order of integration given by $dV = dz\, dy\, dx$. That is, fill in the asterisks in the following expression:

$$\iiint_E f(x, y, z)\, dV = \int_*^* \int_*^* \int_*^* f(x, y, z)\, dz\, dy\, dx.$$

(Optional, for more practice: rewrite this iterated integral using the order $dV = dy\, dz\, dx$.)

Homework 8 for MATH 226, due October 25, 2020

(all numbers indicate exercises from the textbook)

§12.6: 5, 16, 18

§12.7: 12, 24, 28

§13.1: 11–14, 15–18, 24

Additional problems:

1. Using whichever method you like, evaluate the integral

$$I = \iiint_E x\,dV,$$

where E is the region between the cylinders $x^2+y^2=1$ and $x^2+y^2=4$, below the plane $z=y+2$, and above the xy-plane.

2. Convert the following integral to spherical coordinates, then evaluate it:

$$I = \int_{-1}^{0} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} 1\,dz\,dy\,dx.$$

Practice Midterm 2 for MATH 226, section 39559

You have 50 minutes.

Name: **Date:**

Problem	Score
#1	/10
#2	/10
#3	/10
#4	/10
#5	/10
Total	/50

Problem 1 (10 points): Define

$$f(x, y, z) = \frac{1}{x^2} + \frac{4}{y^2} + \frac{9}{z^2}, \tag{1}$$

where $x > 0$, $y > 0$, and $z > 0$. Use Lagrange multipliers to find the minimum value of f on the portion of the surface $x^2 + y^2 + z^2 = 36$ with $x > 0, y > 0, z > 0$. (You may assume that this absolute minimum exists, and that the solution to the system of equations coming from Lagrange multipliers is this absolute minimum.)

Problem 2 (10 points): Consider the function

$$f(x,y) = \frac{1}{5}xy^2 - x \tag{2}$$

on the domain $D = \{(x,y) \mid -2 \leq x \leq 2,\ x^2 \leq y \leq 4\}$.

(a; 3 points) Find the critical points of $f(x,y)$ in the interior of D.

(b; 3 points) Classify the critical points of $f(x,y)$ in the interior of D as local minima, local maxima, or saddles.

Problem 2 (continued).

(c; 4 points) Find the absolute maximum and minimum values of $f(x, y)$ on D.

Problem 3 (10 points): Evaluate the following integrals.

(a; 5 points) $I = \displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{y^3+1}\, dy\, dx$ (switch the order!).

(b; 5 points) $I = \displaystyle\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \sqrt{1+x^2+y^2}\, dy\, dx.$

Problem 4 (10 points): Consider the region E that is under the plane $z + 3y = 16$, inside the cylinder $x^2 + y^2 = 4$, and in the first octant.

(a; 3 points) Rewrite the integral $\iiint_E x\,dV$ as an iterated integral in Cartesian coordinates (i.e. x, y, z). (Do not compute this integral.)

(b; 3 points) Rewrite this integral as an iterated integral in cylindrical coordinates.

(c; 4 points) Evaluate this integral.

Problem 5 (10 points):

(a; 5 points) Rewrite the integral $I = \int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x,y,z)\, dz\, dy\, dx$ as an iterated integral in the order $dx\, dy\, dz$. (You may want to draw a picture of the domain of integration, to help you figure out how to switch the order.)

(b; 5 points) Compute the integral $I = \int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^{1} z\, dz\, dx\, dy.$

Homework 10 for MATH 226, due November 8, 2020

(all numbers indicate exercises from the textbook)

§13.4: 6, 8, 14

§13.5: 2, 10, 12

§13.6: 4, 20, 30, 34

Additional problems:

1. Evaluate (a) directly and (b) using Green's Theorem the line integral $I \coloneqq \int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the counterclockwise-oriented triangle with vertices $(0,0)$, $(2,0)$, and $(2,2)$, and $\mathbf{F}$ is defined by $\mathbf{F}(x,y) \coloneqq \langle 5 - 2xy - y^2, 2xy - x^2 \rangle$.

2. Consider the vector field defined by $\mathbf{F}(x,y,z) \coloneqq \langle yz, xz, xy \rangle$. First, argue that the force field $\mathbf{F}$ is conservative on $\mathbf{R}^3$. Then, find a potential function for $\mathbf{F}$.

3. Let S be the paraboloid defined by $x = \frac{1}{2}\big(y^2 + z^2\big)$, with $0 \leq x \leq 2$. Compute the area of S.

Homework 11 for MATH 226, due November 15, 2020

(all numbers indicate exercises from the textbook)

§13.7: 6, 16, 24

§13.8: 2, 6

§13.9: 6, 12

Additional problems:

1. Consider the line integral $I = \int_C \mathbf{F} \cdot \mathbf{r}$, where

$$\mathbf{F} = \langle x - y, y + x^2, z - 2x^2 \rangle$$

and C is the intersection of the plane $x + 2y + z = 1$ and the coordinate planes, oriented counter-clockwise when viewed from above. Do you think it would be feasible to evaluate I directly? How about using Stokes' theorem? Evaluate I, using whichever method you like.

2. Define E to be the region that is in the first octant and lies inside the sphere $x^2 + y^2 + z^2 = 9$. Compute the flux of the vector field

$$\mathbf{F} = \left\langle \tfrac{1}{3}x^3 + xy^2, \sin(x + z) + yz^2, e^{xy} \right\rangle$$

out of E.