Math 225 Practice Problems

(1) Let A be an $n \times n$ matrix whose null space is $\{\mathbf{0}\}$. If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ span $\mathbf{R}^n$ show that $A(\mathbf{v}_1),\ldots,A(\mathbf{v}_n)$ also span $\mathbf{R}^n$.

(2) Let A be an $n \times n$ matrix satisfying $rk(A) = rk(A^2)$. Show that $\ker(A) = \ker(A^2)$.

(3) Let $V = \mathbf{R}^3$. Let B be the basis $B=\{\mathbf{e}_1+\mathbf{e}_2, \mathbf{e}_1+\mathbf{e}_3, \mathbf{e}_1-\mathbf{e}_3\}$, and C the basis $C=\{2\mathbf{e}_1+\mathbf{e}_2, \mathbf{e}_1-\mathbf{e}_2, 2\mathbf{e}_3\}$.
a) Find the change of basis matrix $P_{C\leftarrow B}$.

b) Find the component vector of $(4,2,1)^T$ with respect to B.

(4). Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 4 \\ 5 & 7 & 16 \end{bmatrix}$.

a) Use elementary row operations to find the reduced row echelon form of A.

b) Find a basis for the row space of A.

c) Find a basis for the image of A.

d) Find a basis for the null space of A.

(5) Let $B=\{x_1, \ldots, x_n\}$ be a basis for the vector space V, and let W be a subspace of V. Does W necessarily have a basis that consists of vectors in B? Carefully explain your answer.

(6) Let A be an n by n matrix, and E its reduced row echelon form. Do A and E necessarily have the same determinant? Carefully explain your answer.

(7) Let V be the vector space of 3×3 matrices with real entries, and let W be the subset of matrices of trace zero. Explain why W is a subspace of V, and find a basis for W.

(8) Let A = $\begin{bmatrix} 3 & 1 & 5 \\ 0 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$. Use elementary row operations to find A^{-1}.

(9) Find all values of a for which the matrix $\begin{bmatrix} 1 & a & 2 \\ 3 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix}$ is non-singular.

(10) Find a set of independent vectors that span the same subspace of $\mathbf{R}^4$ as (1,2,1,2), (1,3,1,4), (5,12,5,14), and (0,2,0,6). Carefully explain your solution.

(11) a) Are the vectors $1+x^2$, $1-x^3$, $x+x^2$, x^2, and $x-2$ a basis for $\mathbf{P}_3$? Why?

b) Do they span $\mathbf{P}_3$? Why?

c) If your answer to (b) is yes find a subset of the vectors that form a basis of $\mathbf{P}_3$.

(12) Find the general solution to

$$\begin{aligned} 2x + 3y + z &= 4 \\ x + y + 2z &= 0 \\ 6x + 8y + 6z &= 8 \end{aligned}$$

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