Math 225 Practice Final Problems

Show all work! Correct answers without supporting work will not be given credit. Calculators and notes are not permitted.

(1) Let $A = \begin{vmatrix} 3 & 2 & 0 & 0 \\ 0 & 4 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 5 \end{vmatrix}$.

a) What are the eigenvalues of A? For each eigenvalue λ find a basis for the λ-eigenspace.

b) Is A diagonalizable. Why?

c) If your answer to (b) is yes find a matrix that diagonalizes A.

(2) Let A be the matrix in problem 1. Find the general solution to $\mathbf{x}'(t) = A\mathbf{x}(t)$. (20 points)

(3) Let A be a 3×3 rank 1 matrix. Is A necessarily diagonalizable? Why?

(4) a) Find the general solution to $(D^2+16)(D-3)^3y=0$.

b) Find the general solution to $(D^2+16)(D-3)^3y=e^{2x}$.

(5) Let T: $V \to W$ be a linear transformation whose null space is (0). If $\{x_1, . . . , x_n\}$ is a basis of V show that the vectors $T(x_1), . . . , T(x_n)$ are part of a basis of W.

(6) Which of the following matrices are diagonalizable? Carefully justify your answer.

a) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 4 & 0 \\ 8 & 2 & 9 \end{pmatrix}$ b) $\begin{pmatrix} 7 & 0 & 0 \\ 2 & 4 & 0 \\ 8 & 1 & 9 \end{pmatrix}$ c) $\begin{pmatrix} 9 & 6 & 5 \\ 0 & 4 & 8 \\ 0 & 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 5 \end{pmatrix}$ e) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 2 & 5 \end{pmatrix}$.

Are (a) ands (c) similar? Why?

(7) Let $A = \begin{bmatrix} 5 & 0 \\ 1 & 5 \end{bmatrix}$.

a) The vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is an eigenvector for A corresponding to the eigenvalue 5. Find a

generalized eigenvector.

b) What is the general solution to $\mathbf{x}'(t) = A\mathbf{x}(t)$?

(8) Find the general solution to the system $\mathbf{x}'(t) = A\mathbf{x}(t)$ where A is the matrix

$$\begin{bmatrix} -1 & 0 & 0 \\ 1 & 5 & 0 \\ 1 & 6 & -2 \end{bmatrix} .$$

(9) Let A be the matrix in problem (8). Find a matrix S such that $S^{-1}AS$ is diagonal.

(10) Let $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$, and let $\mathbf{b} = \begin{bmatrix} e^{2t} \\ e^{t} \end{bmatrix}$.

Find a particular solution to $\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{b}$.

(11) Let V be a vector space, and T: $V \to V$ an isomorphism. If $\{x_1, \ldots, x_n\}$ is a basis of V show that $\{T(x_1), \ldots, T(x_n)\}$ is also a basis of V.

(12) Let T: $\mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation satisfying T(1,1,1) = (1,2,3), T(3,2,1) = (2,1,1), and T(0,0,1) = (3,1,2). What is T(3,3,3)?

(13) T: $\mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation T(a,b,c) = (a+b, a-b, a+c).
Let $B = \{e_1+e_2-2e_3, e_1-e_2, 2e_3\}$, and let $C = \{ e_1-e_3, e_1+e_3, e_1+e_2\}$

a) Find the matrix of T with respect to B.
b) Find the change of basis matrix if we change from the basis B to the basis C.

(14) $A = \begin{vmatrix} 1 & 4 & 6 \\ 0 & 3 & 2 \\ 0 & 0 & 0 \end{vmatrix}$.

a) Is A invertible? Why?

b) What is the rank of A? Why?
c) What is the nullity of A? Why?

(15) Let T be a linear transformation whose matrix is

$$A = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 2 \end{vmatrix} .$$

Using Gaussian elimination compute A^{-1}.

(16) Use variation of parameters to find a particular solution to

$$y'' - 4y' + 5y = e^{2x} \tan x \ , \ 0 < x < \pi/2 \quad .$$