Math 225 Midterm

Show all work, and carefully justify your answers. Correct answers without supporting work will not receive credit. This is a closed book exam. Notes are not permitted.

** Get 2 points EXTRA CREDIT by tagging which problems appear on each page of your file, in gradescope, before clicking submit. **

(1) Let A be an n×n matrix whose null space is {**0**}. If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are independent show that $A(\mathbf{v}_1),\ldots,A(\mathbf{v}_n)$ are also independent. (20 points)

(2) Let $\mathbf{P}_2$ be the vector space of polynomials of degree ≤ 2. Let B be the basis $B=\{1-x^2, 3+2x, 2x^2\}$, and C the basis $C=\{x, x^2, 2\}$. (20 points)

a) Find the change of basis matrix $P_{C\leftarrow B}$.

b) Find the component vector of $4x^2+3x+2$ with respect to B.

(3) Let $A=\begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 5 & 4 & 2 \\ 4 & 9 & 5 & 2 \end{bmatrix}$. (24 points)

a) Use elementary row operations to find the reduced row echelon form of A.

b) Find a basis for the row space of A.

c) Find a basis for the image of A.

d) Find a basis for the null space of A.

(4) Let $V=\mathbf{P}_3$, the polynomials of degree at most 3, and let W be the subset of polynomials p(x) satisfying $p(1) = p(0) = 0$. Explain why W is a subspace of V, and find a basis for W. (20 points)

(5) Find all values of a for which the vectors (a,2,2), (1,2,a), and (1,1,1) are independent. Carefully explain why your answer is complete. (16 points)